BYU ECEn

Parametric Filters For Non-Stationary Interference Mitigation in Airborne Radars

Peter Parker and A. Lee Swindlehurst

Brigham Young University
Dept. of Electrical & Computer Engineering
Provo, UT 84602
voice: (801) 378-4119
fax: (801) 378-6586
email: parkerp@ee.byu.edu

Report Documentation Page

Form Approved OMB No. 0704-0188

1. REPORT DATE **14 MAR 2001**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**
4. TITLE AND SUBTITLE **Parametric Filters For Non Non-Stationary Interference Stationary Interference Mitigation in Airborne Mitigation in Airborne Radars**		5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S) **Peter Parker; A. Lee Swindlehurst**		5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Brigham Young University Dept. of Electrical & Computer Engineering Provo, UT 84602**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) **Defense Advanced Research Projects Agency 3701 North Fairfax Drive Arlington, VA 22203-1714**		10. SPONSOR/MONITOR'S ACRONYM(S) 11. SPONSOR/MONITOR'S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release, distribution unlimited**		
13. SUPPLEMENTARY NOTES **See ADM001263 for entire Adaptive Sensor Array Processing Workshop., The original document contains color images.**		
14. ABSTRACT **See briefing charts.**		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**UU**	**26**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

Motivation: Non-Stationary Interference

- Rapidly changing clutter locus with a circular array or bistatic radar system
- Presence of hot clutter due to an airborne jammer
- Use model of non-stationary interference to derive new filter
- Use small sample support to reduce effect of non-stationary interference

Data Model

- M antennas, N pulses
- Target in primary range bin p

$$\boldsymbol{x}_{\boldsymbol{p}}(t) = b\boldsymbol{a}(\theta)e^{j\omega t} + \boldsymbol{c}_{\boldsymbol{p}}(t), \qquad t = 0, 1, 2, \cdots, N-1$$

spatial steering vector

clutter, jammer, noise, etc.

- Space-Time Slice

$$\boldsymbol{X}_{\boldsymbol{p}} = [\boldsymbol{x}_{\boldsymbol{p}}(0) \quad \boldsymbol{x}_{\boldsymbol{p}}(1) \quad \cdots \quad \boldsymbol{x}_{\boldsymbol{p}}(N-1)]$$

$$= b\boldsymbol{a}(\theta)\boldsymbol{v}^{T}(\omega) + \boldsymbol{C}_{p}$$

temporal steering vector $= \begin{bmatrix} 1 & e^{j\mathbf{w}/T_s} & \cdots & e^{j(N-1)\mathbf{w}/T_s} \end{bmatrix}$

Data Model (cont.)

- Vectorized Forms

$$1.\quad \boldsymbol{x}_p = \mathrm{vec}(\boldsymbol{X}_p)$$
$$= b\,\boldsymbol{v}(\mathbf{w}) \otimes \boldsymbol{a}(\mathbf{q}) + \boldsymbol{c}_p$$

$$2.\quad \boldsymbol{x}_p = \mathrm{vec}(\boldsymbol{X}_p^T)$$
$$= b\,\boldsymbol{a}(\mathbf{q}) \otimes \boldsymbol{v}(\mathbf{w}) + \boldsymbol{c}_p$$

- Secondary Data

target-free range bins

$$\{\boldsymbol{c}_k\}\ k = 1, \cdots, N_s \qquad k \neq p$$

$$E(\boldsymbol{c}_k) = 0 \quad , \quad E(\boldsymbol{c}_k \boldsymbol{c}_k^*) = \boldsymbol{R}$$

interference covariance

Space-Time Autoregressive Modeling

- Define $$\mathrm{H}(z^{-1}) = \sum_{i=0}^{L-1} \mathbf{H}_i z^{-i}$$

M ' x M matrices

- Model: for some *L*,

$$\mathrm{H}(z^{-1})\mathbf{c}_k(t) = \mathbf{H}_0\mathbf{c}_k(t) + \mathbf{H}_1\mathbf{c}_k(t-1) + \cdots + \mathbf{H}_{L-1}\mathbf{c}_k(t-L+1)$$
$$= \varepsilon_k(t)$$

is spatially and temporally white

- To estimate $\mathrm{H}(z^{-1})$, solve

closed-form least-squares solution

$$\min_{\mathbf{H}_0,\cdots,\mathbf{H}_L} \sum_{k=1}^{N_s} \sum_{i=L}^{N} \left\| \mathrm{H}(z^{-1})\mathbf{c}_k(i) \right\|^2$$

Filtering the Primary Data

STAR filter attempts to minimize clutter power:

dimension M'(N-L+1) x 1

$$\varepsilon_k = \begin{bmatrix} H_{L-1} & H_{L-2} & \cdots & H_0 & 0 & \cdots & 0 \\ 0 & \ddots & & & \ddots & & \vdots \\ \vdots & & \ddots & & & \ddots & 0 \\ 0 & \cdots & 0 & H_{L-1} & H_{L-2} & \cdots & H_0 \end{bmatrix} c_k = \mathcal{H} C_k$$

Span($\mathcal{H}$) orthogonal to clutter subspace if it dominates white noise:

$$\mathbf{R} = \underbrace{H^{\perp} \mathbf{Q} H^{\perp *}}_{\text{clutter \& jamming}} + \underbrace{\sigma^2 \mathbf{I}}_{\text{white (thermal) noise}}$$

so we project onto the orthogonal subspace using a matched subspace filter:

$$\mathbf{x}'_p = H^* \underbrace{\left(H H^*\right)}_{\text{banded block Toeplitz}}^{-1} H \, \mathbf{x}_p$$

Algorithm Summary

1. Use SVD on secondary data to solve for $[\mathbf{H}_0 \quad \mathbf{H}_1 \quad \cdots \quad \mathbf{H}_{L-1}]$

   computational order: $O\left(N_s M^2 L^2 (N - L + 1)\right)$

2. Form $\mathcal{H}$ and filter data: $\mathbf{P}_{\mathcal{H}^*}\mathbf{x}_p$

   computational order: $O\left(M' M^2 L^2 (N - L + 1)\right)$

3. Perform regular beam and Doppler filtering for detection

   computational order: negligible

Resultant test statistic is $(\mathbf{v} \otimes \mathbf{a})^* \mathbf{P}_{\mathcal{H}^*}\mathbf{x}_p$ not $(\mathbf{v} \otimes \mathbf{a})^* \mathbf{R}^{-1}\mathbf{x}_p$

Prior Work

- Vector AR models used previously for clutter modeling by Michels, Rangaswamy, etc.
- Standard STAP filters extended to handle range-varying and hot clutter models by Zatman, Rabideau, etc.
- Matched subspace detectors used for subspace interference by Scharf
- Here, we extend the parametric model to handle the non-stationary interference

Range-Varying STAR Filter

- To improve performance at short ranges, use linearly varying matrix coefficients:

extended data vector

$$\sum_{i=0}^{L-1} \left[\boldsymbol{H}_i \quad \Delta \boldsymbol{H}_i \right] \begin{bmatrix} \boldsymbol{c}_k(t-i) \\ \alpha k \boldsymbol{c}_k(t-i) \end{bmatrix} = \varepsilon_k(t), \quad t = L+1, \cdots, N$$

M' X M matrices

spatially and temporally white

- Analogous to ESMI technique of Hayward

- To normalized the noise subspace

$$\alpha = \sqrt{\frac{12}{(N_s+2)(N_s+1)}}$$

Range-Varying STAR Filter

- Minimize clutter power assuming linearly varying statistics

$$e_k = \tilde{\mathcal{H}} \begin{bmatrix} \boldsymbol{c}_k \\ \alpha k \boldsymbol{c}_k \end{bmatrix}$$

where

Extended STAR filter coefficients $\Delta\mathcal{H}$

$\mathcal{H}$ from STAR filter

$$\tilde{\mathcal{H}} = \left[\begin{array}{cccccc|cccccc} \boldsymbol{H}_{L-1} & \cdots & \boldsymbol{H}_0 & & \boldsymbol{0} & & \Delta\boldsymbol{H}_{L-1} & \cdots & \Delta\boldsymbol{H}_0 & & \boldsymbol{0} & \\ & \ddots & & \ddots & & & & \ddots & & \ddots & & \\ \boldsymbol{0} & & \boldsymbol{H}_{L-1} & \cdots & \boldsymbol{H}_0 & & \boldsymbol{0} & & \Delta\boldsymbol{H}_{L-1} & \cdots & \Delta\boldsymbol{H}_0 & \end{array}\right]$$

- Filter data with matched subspace filter

k=0 for primary range bin

$$\mathbf{x}'_p = \tilde{\mathcal{H}}^* \left(\tilde{\mathcal{H}}\tilde{\mathcal{H}}^*\right)^{-1} \tilde{\mathcal{H}} \begin{bmatrix} \mathbf{x}_p \\ \mathbf{0} \end{bmatrix} = \boldsymbol{P}_{\tilde{\mathcal{H}}^*} \begin{bmatrix} \mathbf{x}_p \\ \mathbf{0} \end{bmatrix}$$

Range-Varying STAR Filter

- Define

$$\boldsymbol{C}_k = \begin{bmatrix} \boldsymbol{c}_k(L+1) & & \boldsymbol{c}_k(N) \\ \vdots & \cdots & \vdots \\ \boldsymbol{c}_k(1) & & \boldsymbol{c}_k(N-L) \end{bmatrix}$$

- Estimate filter coefficients:

$$[\boldsymbol{H}_0 \quad \cdots \quad \boldsymbol{H}_{L-1} \; \Delta\boldsymbol{H}_0 \quad \cdots \quad \Delta\boldsymbol{H}_{L-1}]$$

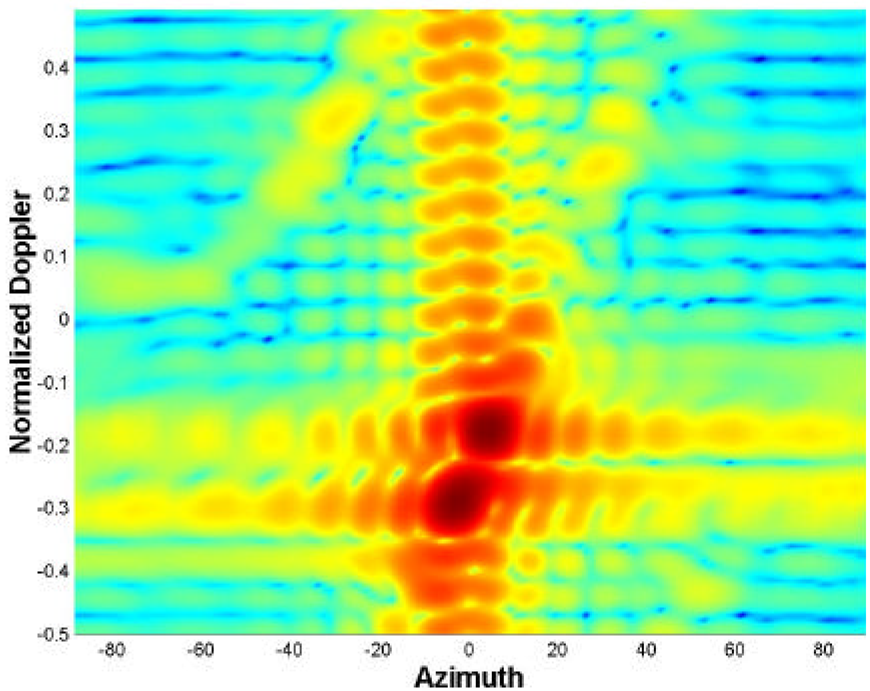
as the left singular vectors with the *M'* smallest singular values of the extended data matrix:

$$\begin{bmatrix} \boldsymbol{C}_{-\boldsymbol{N_s}/2} & \cdots & \boldsymbol{C}_{\boldsymbol{N_s}/2} \\ -\dfrac{\alpha N_s}{2}\boldsymbol{C}_{-\boldsymbol{N_s}/2} & \cdots & \dfrac{\alpha N_s}{2}\boldsymbol{C}_{-\boldsymbol{N_s}/2} \end{bmatrix}$$

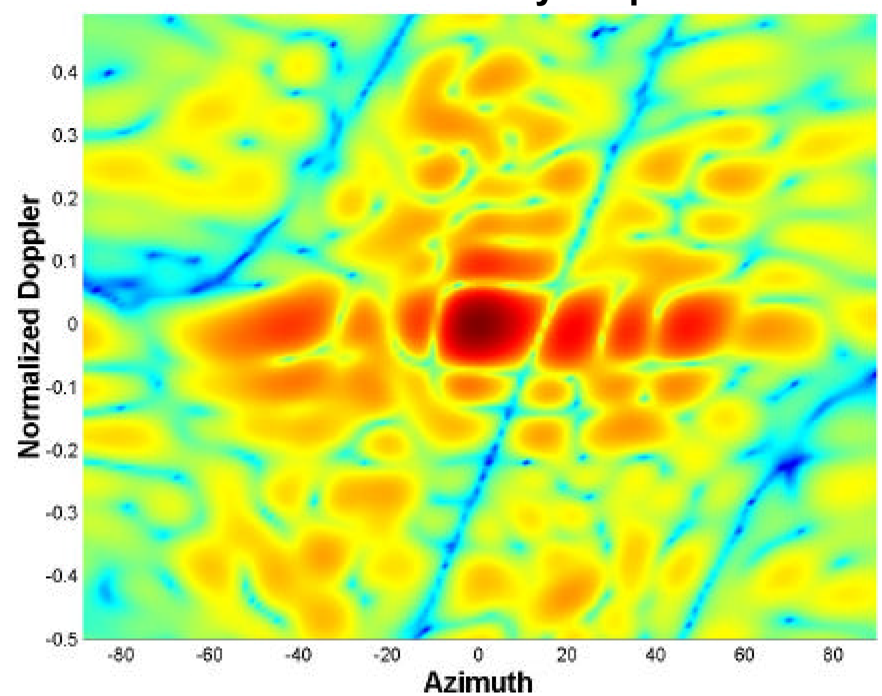
ESTAR Filter Example

20 element circular array, 18 pulses
SCR = -58dB   SNR = 10dB

Primary data vector snapshot at 20 km



4 tap ESTAR filter,
20 secondary snapshots



Computational Comparison

Some typical numbers: $M = 20$, $N = 18$, $M' = 20$

- STAR Filter (L=5): $O(140{,}000N_s) + O(2{,}800{,}000)$

- ESTAR Filter (L=4):

$$O\left(4N_sM^2L^2(N-L+1)\right) + O\left(M'M^2L^2(N-L+1)\right)$$
$$= O(384{,}000N_s) + O(1{,}920{,}000)$$

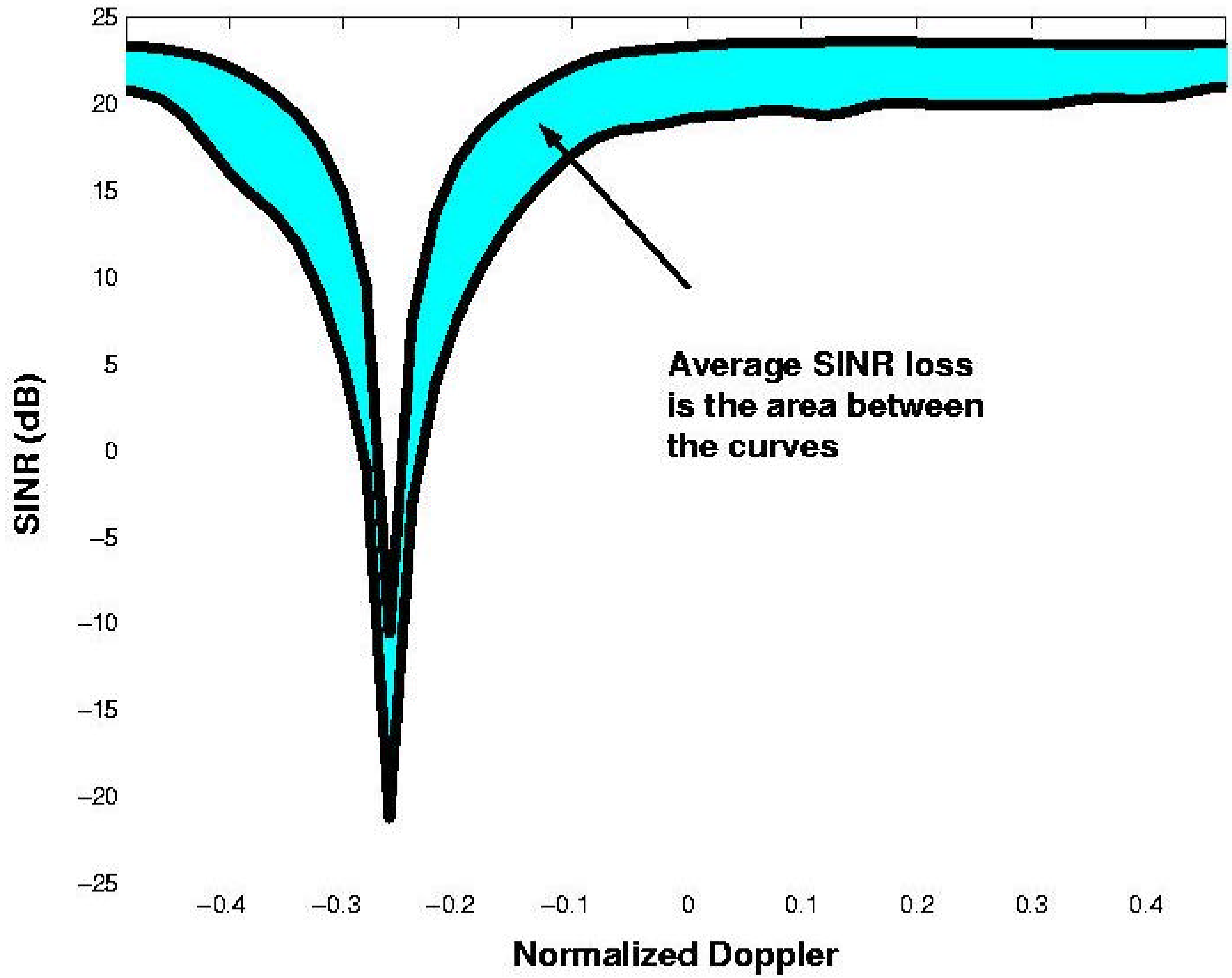
- Extended PRI staggered algorithm:

$$O(4N_sM^2K^2(N-K+1)) + O\left(4\rho M^2K^2(N-K+1)\right) = O(230{,}000N_s) + O(20{,}000{,}000)$$

# of sub-CPIs = 3

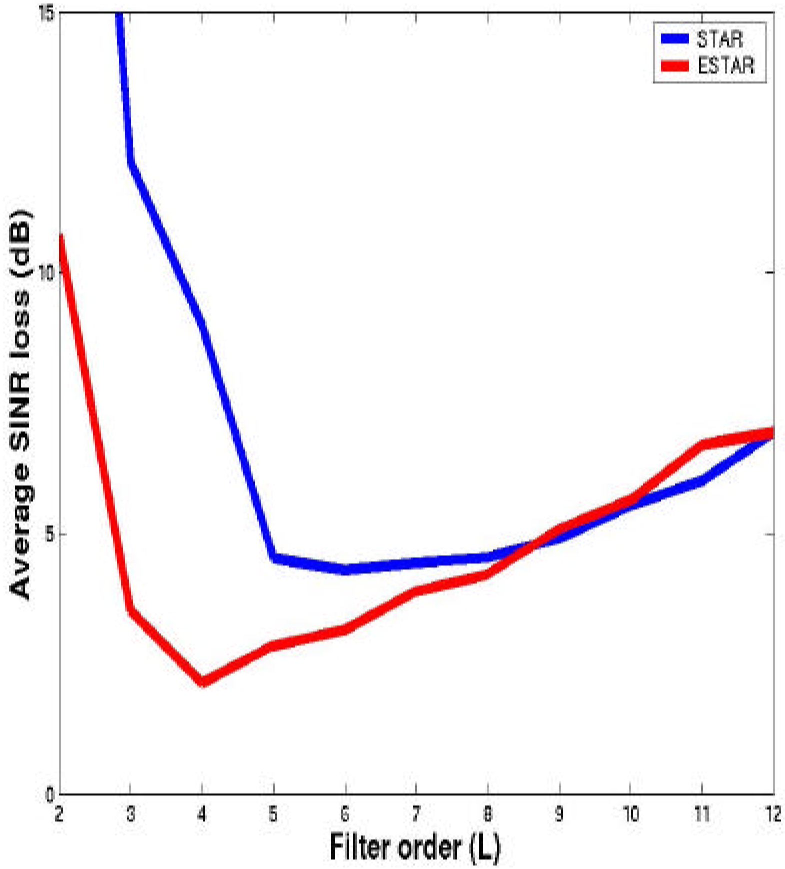
rank of sub-CPI covariance $\cong 90$

Average SINR Loss

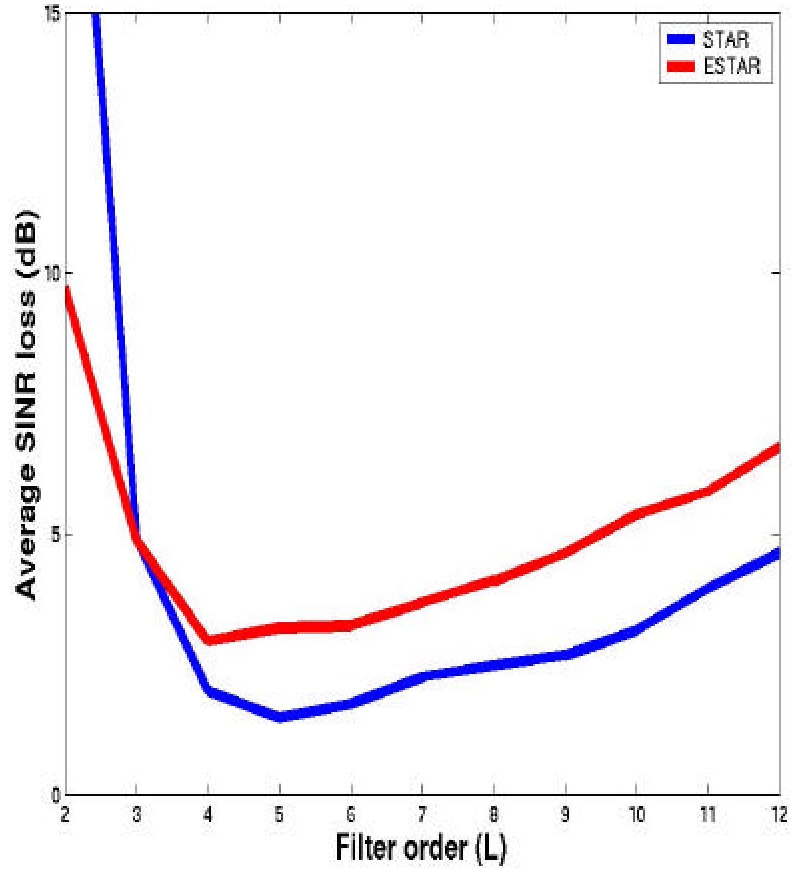


Performance with Range-Varying Weights

20 km range



30 km range

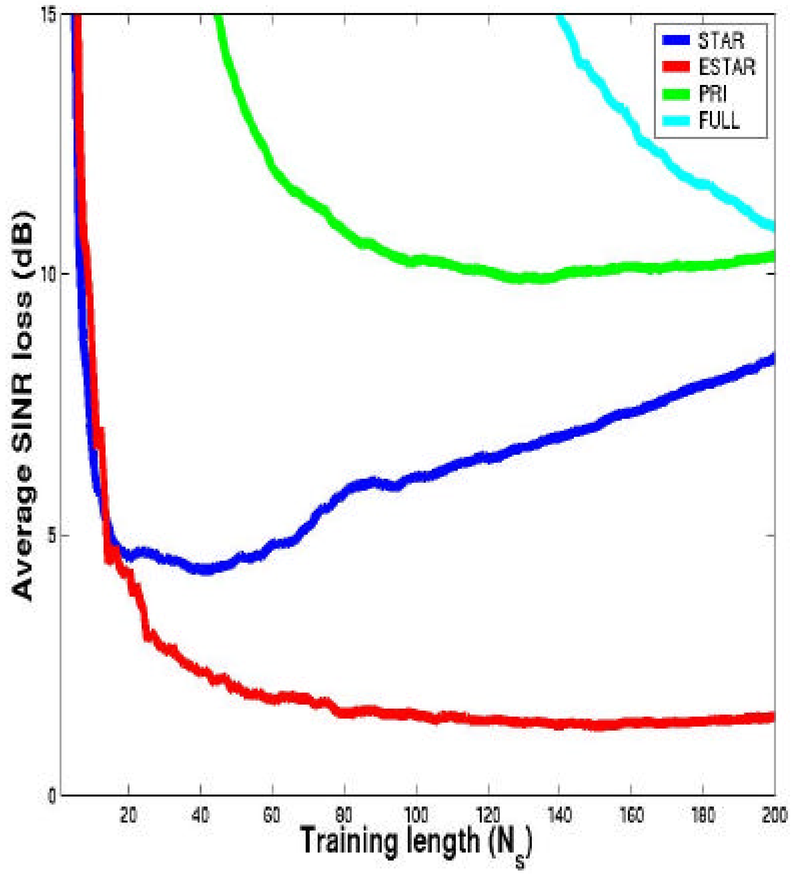


N_s=50 training vectors – 2 km training window

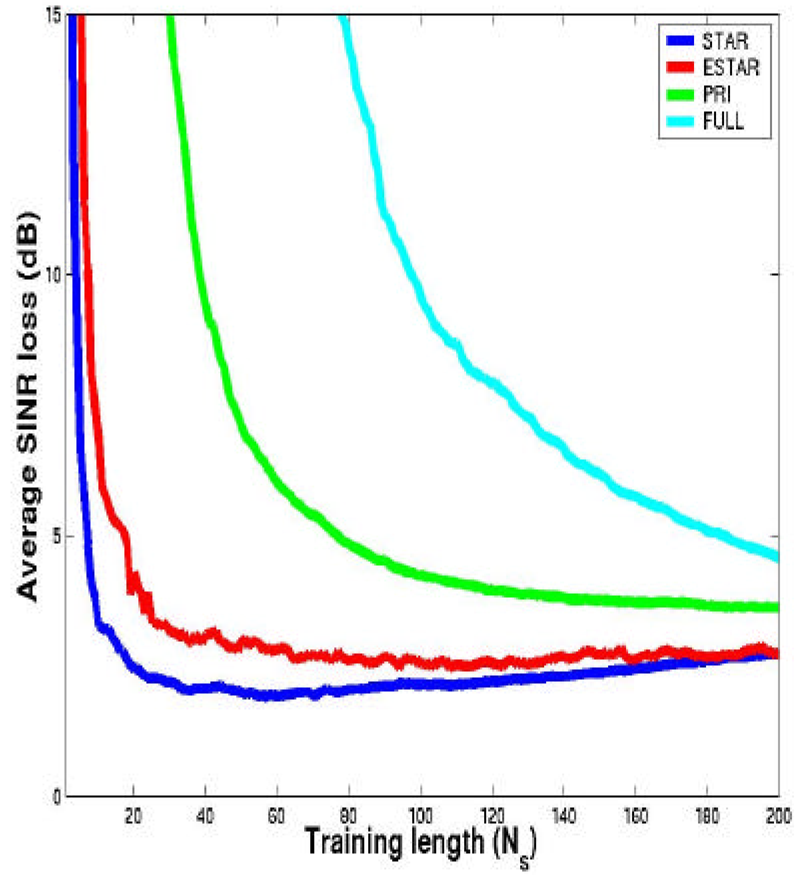
BYU ECEn

Performance with Range-Varying Weights

20 km range



30 km range



L=5 for STAR filter – L=4 for ESTAR filter

3-D STAR Filter for Hot Clutter

- Update filter for each new pulse received
    - Derive slow-time varying STAR filter
    - Can be used with intrinsic clutter motion
- Add fast-time matrix taps to exploit correlations across range bins
    - Additional filter taps help mitigate mainbeam jamming signals

Slow Time-Varying STAR Filter

- Same structure as the STAR filter but with new coefficients for each pulse

$$\mathcal{H}_{TV} = \begin{bmatrix} \boldsymbol{H}_{L-1}(1) & \cdots & \boldsymbol{H}_0(1) & & \boldsymbol{0} \\ & \ddots & & \ddots & \\ \boldsymbol{0} & & \boldsymbol{H}_{L-1}(N-L+1) & \cdots & \boldsymbol{H}_0(N-L+1) \end{bmatrix}$$

- Additional sample support required due to additional parameters to model slow-time variation

3D-STAR Filter

- Use slow-time varying STAR filter to model correlation across pulses

- For some fast-time filter order *J*, model the fast-time correlation as:

$$\sum_{j=0}^{J-1} \mathcal{H}_{TV,j} c_{k-j} = \varepsilon_k , \quad k = J+1,\cdots ,P$$

subscript denotes which fast-time sample $\mathcal{H}$ is associated with

number of fast-time samples used to whiten data

- Similar to a 2-D vector AR model with the slow-time taps changing with each pulse

Estimation of Parameters

- Define

$$\widetilde{\boldsymbol{H}}(t) = \left[\boldsymbol{H}_{0,0}(t) \quad \boldsymbol{H}_{1,0}(t) \quad \cdots \quad \boldsymbol{H}_{L-1,J-1}(t)\right]$$

$$\boldsymbol{g}_k(t) = \begin{bmatrix} \boldsymbol{c}_k(t+L-1) \\ \vdots \\ \boldsymbol{c}_k(t) \end{bmatrix} \qquad \boldsymbol{G}_k(t) = \begin{bmatrix} \boldsymbol{g}_{k+J-1}(t) & & \boldsymbol{g}_{k+P-1}(t) \\ \vdots & \cdots & \vdots \\ \boldsymbol{g}_k(t) & & \boldsymbol{g}_{k+P-J}(t) \end{bmatrix}$$

- Least squares solution:

$$\min_{\widetilde{\boldsymbol{H}}(t)} \sum_{k=1}^{N_s} \left\| \widetilde{\boldsymbol{H}}(t)\boldsymbol{G}_{\mathrm{k}}(t) \right\|^2$$

- New minimization for each slow-time step

Filtering the Primary Data

- 3D-STAR filter can be written as:

$$
\mathrm{H} = \begin{bmatrix} \mathcal{H}_{TV,J-1} & \cdots & \mathcal{H}_{TV,0} & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathcal{H}_{TV,J-1} & \cdots & \mathcal{H}_{TV,0} \end{bmatrix}
$$

- Project out the interference using 3D matched subspace filter

$$
\begin{bmatrix} \mathbf{x}'_p \\ \vdots \\ \mathbf{x}'_{p-P+1} \end{bmatrix} = \mathrm{H}^{*}(\mathrm{HH}^{*} \quad \mathrm{H} \begin{bmatrix} \mathbf{x}_p \\ \vdots \\ \mathbf{x}_{p-P+1} \end{bmatrix} = \mathrm{P}_{\mathrm{H}} \cdot \begin{bmatrix} \mathbf{x}_p \\ \vdots \\ \mathbf{x}_{p-P+1} \end{bmatrix}
$$

Highly structured nature of subspace, small sample support make full 3-D STAR solution feasible

Computational Comparison

Some typical numbers: M = 20, N = 18, M' = 20, P = 3

- STAR Filter (L=7): $O(235{,}000N_s) + O(4{,}700{,}000)$
- 3D-STAR Filter (L=2): $O\left(N_s (MLJ)^2 (N-L+1)(P-J+1)\right)$ $+\,O\left(M' (MLJ)^2 (N-L+1)(P-J+1)\right)$
- J=2: $O(218{,}000N_s) + O(4{,}350{,}000)$
- J=1: $O(82{,}000N_s) + O(1{,}630{,}000)$
- Optimized 3D-post-Doppler algorithm:

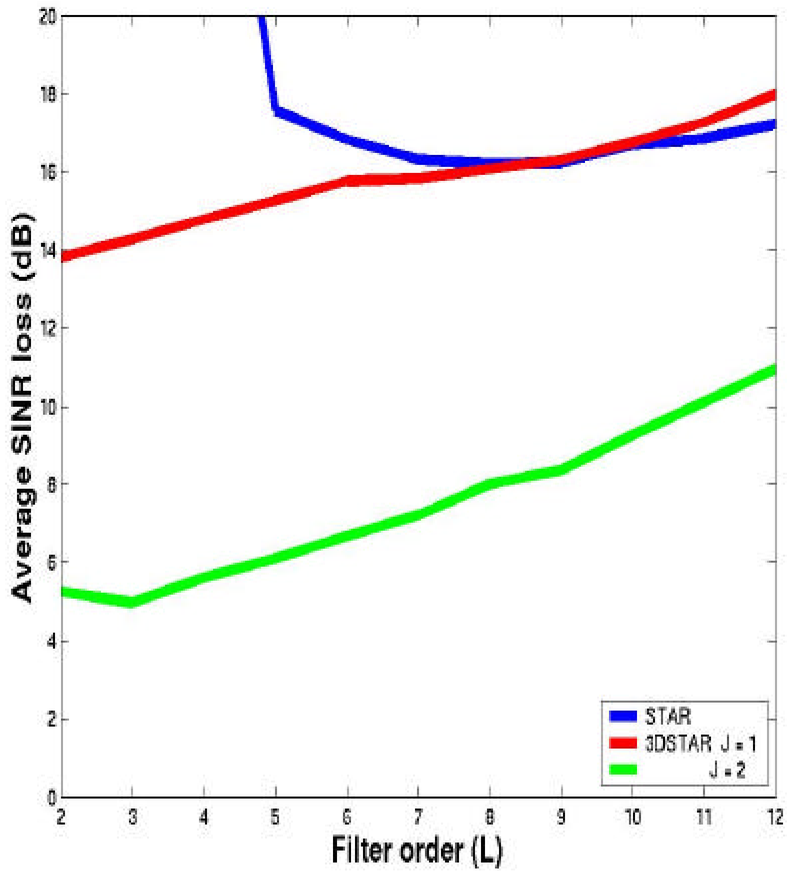
$$O\left(N_s (MKP)^2 (N-K+1)\right) + O\left(\tilde{n}(MKP)^2 (N-K+1)\right) = O(518{,}000N_s) + O(70{,}000{,}000)$$

# of sub-CPIs = 3

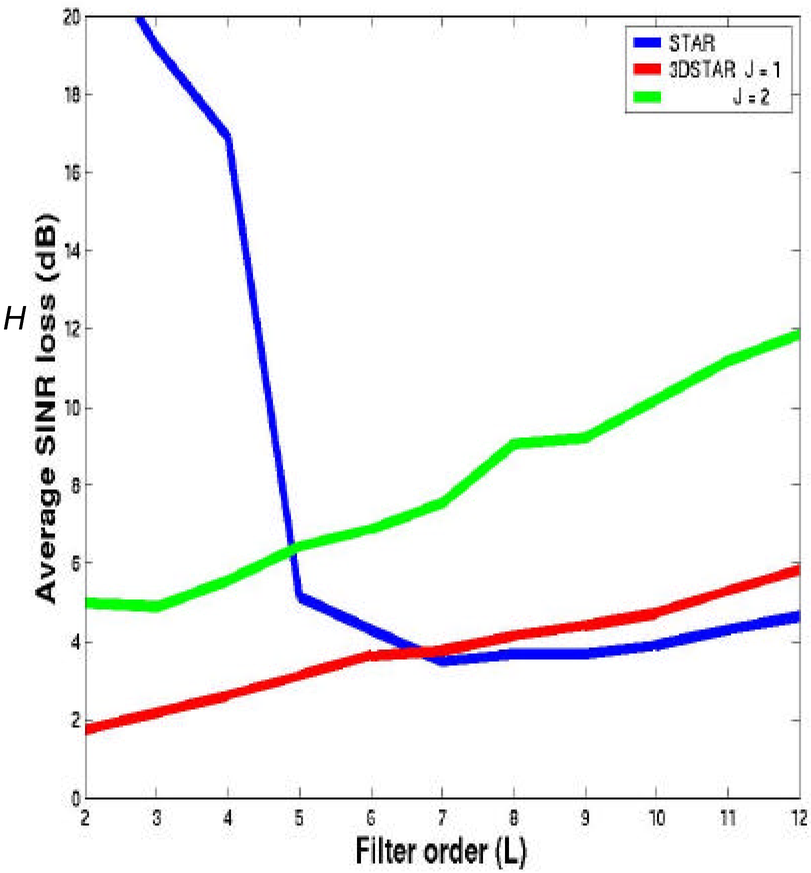
rank of sub-CPI covariance $\cong 135$

Hot Clutter Examples

Direct path jamming signal in mainbeam: JDOA=1º



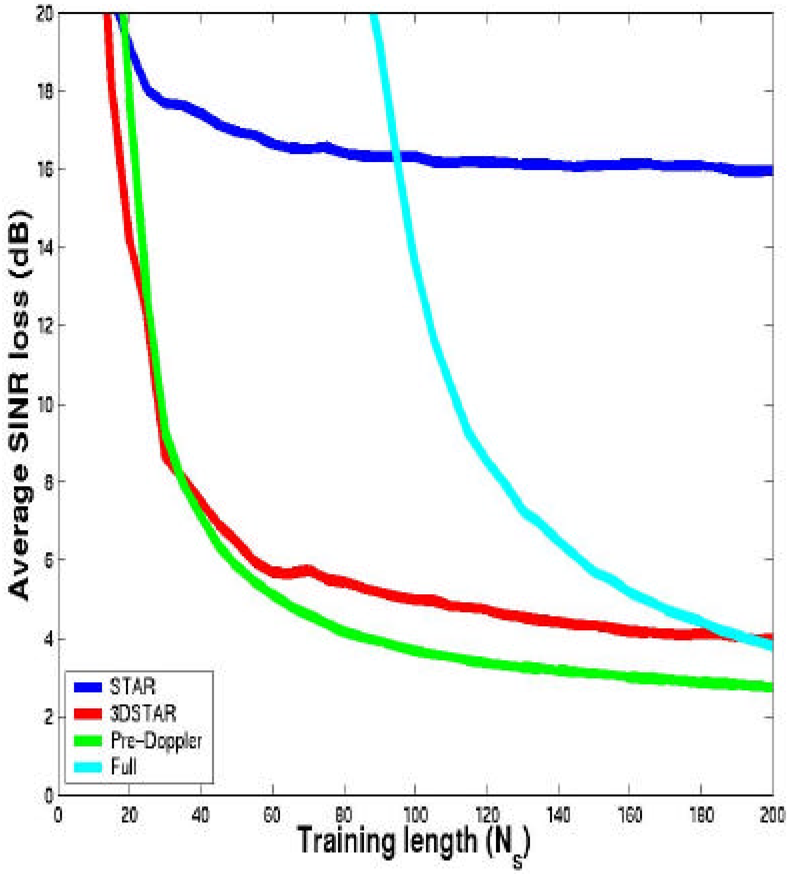
Only multipath component in mainbeam: JDOA=-20º

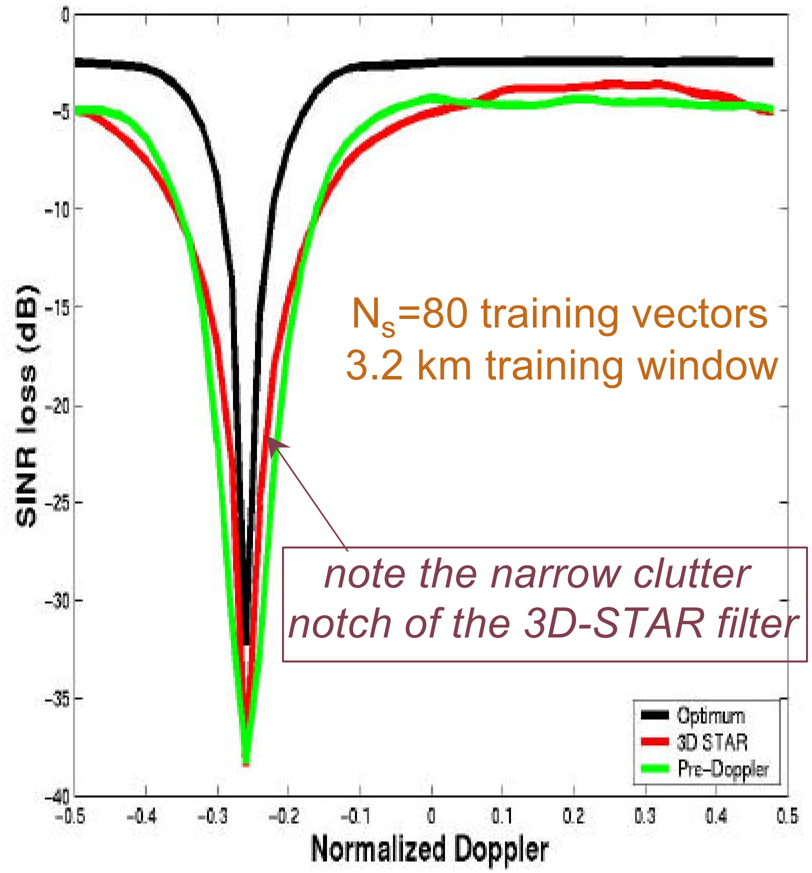


N_s=100 training vectors – 4 km training window

Hot Clutter Examples

Direct path JDOA=1°

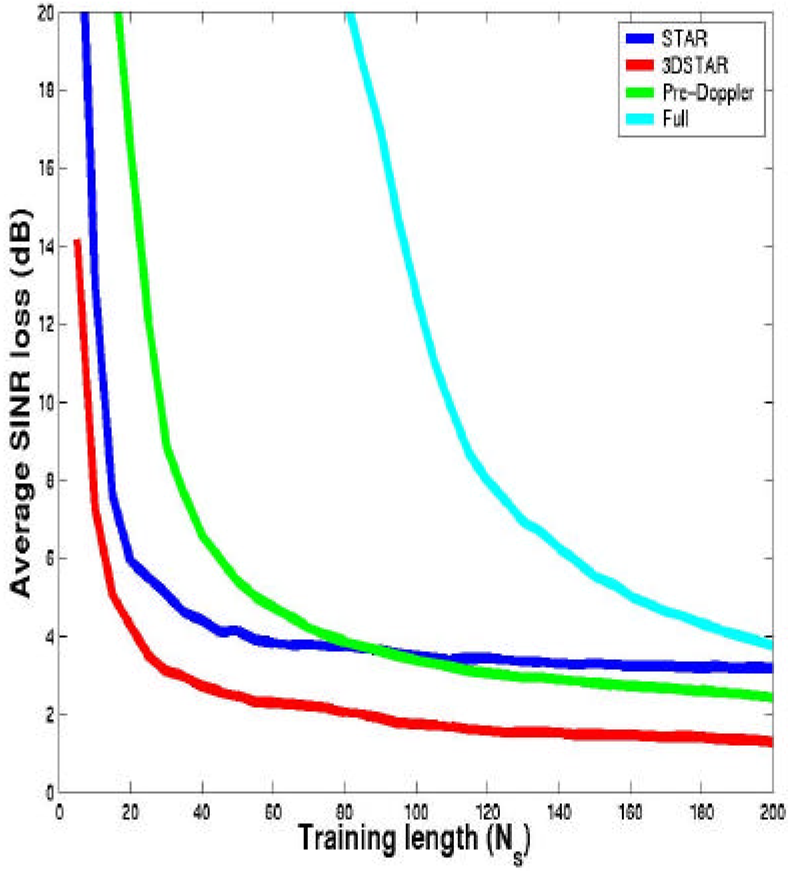


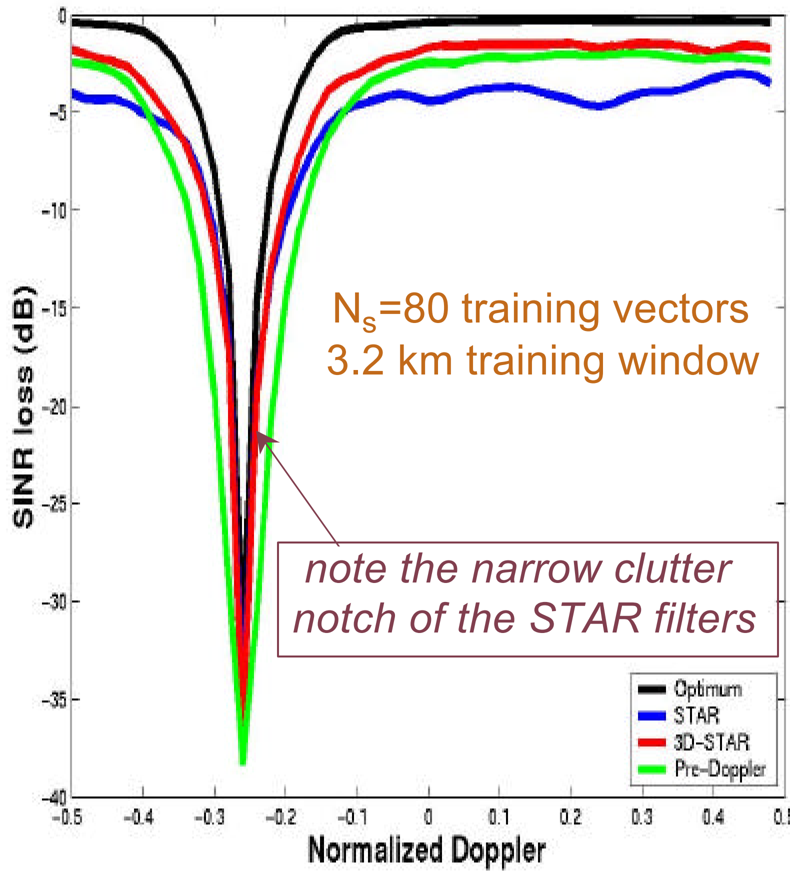


L=2, J=2 for 3D-STAR filter – L=7 for STAR filter

Hot Clutter Examples

Direct path JDOA=-20°





L=2, J=1 for 3D-STAR filter – L=7 for STAR filter

BYU ECEn

Conclusions

- STAR based filtering ideal for STAP problems that require small secondary sample support
- Easily extended to handle hot or range-varying clutter models
- Simulations with realistic circular array data show promising performance
- The structured nature of the filters leads to computationally efficient algorithms